University of Bahrain

College of Information Technology
Department of Computer Science

ITCS253 Discrete Structures II

Second Semester 2014/2015

Exam #1 – 75 Minutes

STUDENT NAME	
STUDENT#	
SECTION	
SERIAL	

This exam contains **6** pages (including this cover page) and **6** questions. Check to see if any pages are missing. Enter all requested information.

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Question	Points	Score
1	7	6
2	7	7
3	8	5
4	4	4
5	7	7
6	7	7
Total:	40	36

Instructor: Dr. Ali Alsaffar   Sections# 1 & 2

Answer all questions

(1) Let $f : \mathbf{A} \to \mathbf{R}$ defined by $f(x) = \dfrac{\sqrt{x-1}}{x-2}$ and $g : \mathbf{R} \to \mathbf{R}$ defined by $g(x) = |x| - 1$.

(a) [**2 points**] Find an appropriate domain **A** for f. Justify your answer.

In domain of A it's must $X \geq 3$ because it's impossible to the value under the root be negative and also it's not exist the $X = 2$ because the dominitor will be Zero so the domain will be like that domain $= \{ \sqrt{2}, \frac{\sqrt{3}}{2}, \frac{2}{3}, \frac{\sqrt{3}}{4}, \dots \}$

2

(b) [**3 points**] Find the range of g.

$$g(x) = \begin{cases} X \cancel{+}1 & X \geq 0 \\ -X \cancel{+} 1 & X < 0 \end{cases}$$

In case: $X \geq 0$

let $y = X \cancel{+} 1$, solve for X

$X = y + 1$

$y + 1 \geq 0 \Rightarrow y \geq -1$

Range $= \{ y \in R \mid y \geq 1 \}$

2.5

In Case 2: $X < 0$

let $y = -X + 1$, solve for X

$-X = y - 1 \Rightarrow X = -y + 1 < 0 \Rightarrow y > 1$

$\cancel{X \leq}$ $-y + 1 < 0 \Rightarrow -y < -1 \Rightarrow y < 1$

Range $= \{ y \in R \mid y < 1 \}$

(c) [**2 points**] (i) Find $g \circ f$ (ii) Does $f \circ g$ exist? Justify your answer.

~~(i) $(g \circ f)(x) = g(f(x)) = gX$~~

(i) ~~(g∘f)~~ In case: $x \geq 0$

$(g \circ f)(x) = g(f(x)) = g\left(\frac{\sqrt{X-1}}{X-2}\right) = \left|\frac{\sqrt{X-1}}{X-2}\right| - 1$

In case: $X < 0$

$(g \circ f)(x) = g(f(x)) = g\left(\frac{\sqrt{X-1}}{X-2}\right) = \frac{-\sqrt{X-1}}{X-2} - 1$

1

(ii) Doesn't exist because, not all element in $g(x)$ ~~is~~ in $f(x)$

1/2

(2) Define a function $f : \mathbf{R} \to \mathbf{R}$ by $f(x) = 3x^2 + 12x + 2$.

(a) **[4 points]** Find the range of f.

let $y = 3x^2 + 12x + 2$, solve for x

$3x^2 + 12x + 2 - y = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 3$, $b = 12$, $c = 2 - y$

$$x = \frac{-12 \pm \sqrt{(12)^2 - 4(3)(2-y)}}{2(3)}$$

4

$$= \frac{-12 \pm \sqrt{144 - (12)(2-y)}}{6}$$

$$= \frac{-12 \pm \sqrt{144 - 24 + 12y}}{6} = \frac{-12 \pm \sqrt{120 + 12y}}{6}$$

$120 + 12y \geq 0 \Rightarrow -12y \geq +120$

$\Rightarrow y \geq \frac{+120}{-12}$

$\Rightarrow y \geq -10$

$\therefore$ Range $= \{y \in R \mid y \geq 10\}$

(b) **[3 points]** Is f one-to-one ? Prove your answer.

let $x_1, x_2 \in R$ and assume $f(x_1) = f(x_2)$

$3x_1^2 + 12x_1 + \cancel{2} = 3x_2^2 + 12x_2 + \cancel{2}$

$3x_1^2 + 12x_1 = 3x_2^2 + 12x_2$

$3x_1^2 - 3x_2^2 = -12x_1 + 12x_2$

$3(x_1^2 - x_2^2) = -12(x_1 - x_2)$

$3(x_1 - x_2)(x_1 + x_2) = -12(x_1 - x_2)$

3

Case 1: $x_1 = x_2$ we are done (one-to-one)

Case 2: $x_1 \neq x_2$

$3(x_1 - \cancel{x_2})(x_1 + x_2) = -12(x_1 - \cancel{x_2})$

$x_1 + x_2 = \frac{-12}{3} = -4$

assume $f(x_1) = f(x_2)$ but $x_1 \neq x_2$ for example

$x_1 = -8$, $x_2 = 4$, then $x_1 + x_2 = 4$

$-8 + 4 = -4$

but $x_1 \neq x_2$ so ~~f isn't one~~

$\therefore$ f isn't one-to-one function.

(3) Consider the operation * defined on $G = \{1, 2, 3, 4\}$ by the table.

*	1	2	3	4
1	3	4	1	2
2	4	1	2	3
3	1	2	3	4
4	2	3	4	1

Assume that * is an *associative* operation on G. Please answer the following questions and justify your answers.

(a) **[5 points]** Show that $(G, *)$ is a group.

~~It's~~ closed with G?
1, 2, 3, 4 ∈ G (all enteries in matrix is element ∈ in G)
∴ It's closed with G

1

Identity element?
left Identity element:
~~e * a =~~ let a, e ∈ G and a = 1 and e = 3 (from Table)
~~e~~ e * a = a ⇒ 3 * ~~1~~ = 1

1/2

Right Identity element:
let a, e ∈ G ~~e=3~~ and a = 1, ~~B~~ = 3
a * e = a = ~~1~~ * 3 = 1
∴ Theres is identity element. e = ?

(Note): Inverse in the ~~bace~~ back of Page

(b) **[1 point]** Show that * is commutative on G.

By looking to left diagnal we ~~g~~know the table is symmetric, so if ~~its~~ it's is symmetric then it's commutative.

1

(c) **[2 points]** Let $x \in G$ and suppose $(2 * x)^{-1} = 3$. Find x.

$(2 * x)^{-1} = 3$

$(2 * x^{~~-1~~})^{-1} = x^{-1} * 2^{-1} = 3$ (Inverse of 2 is 4)

$= x^{-1} * 4 = 3$

from table, the number how give us 3 ~~with~~ when we check is any number ~~give~~ * with ~~E~~ ~~a~~ invers of 2 is 2

$\therefore x^{-1} = 2$, by show the table we know the $x = 4$

2

Inverse ?

let $a, a^{-1} \in G$ and $a = 1$, $a^{-1} = 1$ (from Table)

. left Inverse :

$a^{-1} * a = e \Rightarrow \cancel{3*} a^{-1} * a = 3 \Rightarrow 1 * 1 = 3$

1/2 . Right Inverse

$a * a^{-1} = e \Rightarrow a \cancel{3} * a^{-1} = 3 \Rightarrow 1 * 1 = 3$

$4^{-1} = ?$

$2^{-1} = ?$

$\cancel{3}$ ∴ It's has inverse

~~B~~ from previous information in question ~~we know that~~ (associative)

and what we find (Closed with G , Identity element , Inverse)

G is group .

~~Q3: Part c: complete the answer~~

~~∴ 3 is identity element of group G~~

∴ X

(4) **[4 points]** Suppose G is a group. For any $a, b \in G$, if $a^{-2} = e$, show that

$$[(a*b)^{-1}*b]^2 = e$$

$$[(a*b)^{-1}*b]^2 = [(a*b)^{-1}*b]*[(a*b)^{-1}*b]$$
$$= [b^{-1}*a^{-1}*b]*[b^{-1}*a^{-1}*b]$$
$$= [b^{-1}*a^{-1}*b*b^{-1}*a^{-1}*b]$$
$$= [b^{-1}*a^{-1}*\underbrace{(b*b^{-1})}^{e}*a^{-1}*b]$$
4
$$= [b^{-1}*\underbrace{(a^{-1}*e)}^{a^{-1}}*a^{-1}*b]$$
$$= [b^{-1}*\underbrace{(a^{-1}*a^{-1})}^{a^{-2}}*b]$$
$$= [b^{-1}*\underbrace{a^{-2}}^{e}*b]$$
$$= [(b^{-1}*e)*b] = [b^{-1}*b] = e$$

(5) Answer the following questions.

(a) **[3 points]** Consider the sequence of numbers $3, 2, 1, 0, -1, -2, -3, -4, \ldots$.
Write a recurrence relation to the above sequence.

3 $a_0 =$ 3, $a_n =$ $a_0 - 1$, $n \geq$ ~~0~~ 1

(b) **[4 points]** Find a solution to the recurrence relation in (a) using the *Iteration Method.*

$a_1 = a_0 - 1$
$a_2 = a_1 - 1 = a_0 - 1 - 1 = a_0 - 2$
$a_3 = a_2 - 1 = a_0 - 1 - 1 - 1 = a_0 - 3$
$a_4 = a_3 - 1 = a_0 - 1 - 1 - 1 - 1 = a_0 - 4$
$\vdots$

4 ~~$a_n = a_0 - (n-1) = a_0 - n + 1$~~
~~$= 3 - n + 1$~~
$a_n = a_0 - n$
$= 3 - n$